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Closed-loop Identification of a Multivariable Dynamic Knee Rig

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Abstract: This paper presents the closed-loop identification of a multivariable dynamic knee rig, which is used to evaluate the performance of newly developed knee prosthesis. Unlike standard dynamic knee rigs, the system used in this research has the capability to impose cyclic movements on the knee joint, making it a multivariable system, i.e. multi-input/multi-output (MIMO) system. To obtain a simplified model of the system, interaction between the multivariable outputs is neglected. As the system is unstable in open-loop, an indirect closed-loop identification method is used to construct a model of the system. Parametric identification of the closed-loop system is performed using a specific version of the prediction error method: the ARMAX method. For each of the obtained models of the multivariable system the normalized root-mean-square error (NRMSE) value was obtained which quantifies the goodness of fit of the model compared to the experimental data. Based on the closed-loop models, a model for the open-loop system was derived and validated against the measured system's response. The results indicate that the identified models correspond to the measured signals and validation is obtained. However, improvements to the models are possible when taking into account the interaction between the variables of the MIMO system which will be the focus of future research. Future work also consists of developing an advanced controller based on the identified models to control the system's outputs.

Keywords: Closed-loop identification, Prediction error methods, ARMA parameter estimation, Multi-input/multi-output systems, Control, Biomedical systems.

1. INTRODUCTION

The most complex joint of the human body is the knee joint as it allows movement between three bones and transfers large loads from the upper to the lower part. Consequently, the knee joint is prone to injuries and new treatment options or preventive measures are required daily (Pitkin (2010)). Therefore, understanding the intricate knee joint biomechanics has been the focus of many investigations (Maquet (1984); Chevalier (2014)).

The cause of knee injuries is twofold. Firstly, demographics show considerable aging of the population (OECD (2014)). In an aging population, wear and tear of the knee joint cartilages and ligaments are the main cause of an increased number of knee injuries. Secondly, statistical analysis of sport injuries indicates that 37% of all sport injuries are accounted for by knee injuries (Majewski (2006); De Loës (2000)).

For many patients with severe knee injuries, the only course of treatment is eventually a total knee replacement (TKR) where the natural knee joint is replaced by a knee prosthesis. As a result of the aging population, the amount of knee replacing operations performed in the EU has increased by more than 25% in the last few years (OECD (2012)). Research into optimization of knee prosthetics is financially sound when looking at the average cost of 9000 Eur per TKR procedure in the EU (Surgery Price (2013)).

To evaluate the performance of newly designed knee prosthetics and gain insight into the intricate knee dynamics, a biomedical system called a dynamic knee rig, is used by orthopedic surgeons. The main idea behind a dynamic knee rig is to impose natural movements onto post-mortem knees or mechanical knees in order to gain insight into their biomechanics. The first reported version of a dynamic knee rig was the knee joint simulator developed by Shaw and Murray (1973). Over the years it transformed into a dynamic knee rig which can impose squat movements onto knee joints (Bourne (1978)). A squat movement is

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medically relevant to investigate because of its occurrence in daily movements such as rising from a chair (Zavatsky (1997)). However, as in revalidation many cyclic motions are performed, a new version of the dynamic knee rig was developed which has the possibility to impose a cyclic motion onto the knee joint using multiple actuators making it a multi-input/multi-output (MIMO) system but also to extend towards other clinically relevant motion patterns. The novel aspect of this simulator lies withing the freedom in the sagittal plane i.e. the plane dividing the human body in left and right. A suitable mathematical model for this kind of system has not been determined previously and makes the objective of our work presented in this paper.

Identification techniques use measured input-output data to obtain a mathematical model of dynamical systems. Both parametric and non-parametric identification techniques can be used to obtain a system's model (Pai (2013)). Parametric identification results in a parametric model which can be used as a basis for controller design. The prediction error method (PEM) is a parametric identification technique which has a close link to the Maximum Likelihood method (Fisher (1912)) and was first used for the estimation of dynamical models by Box and Jenkins (1970). For stable systems, PEM can be used to identify the mathematical model of the system itself by using input-output data of the system, i.e. open-loop identification. However, for unstable systems, such as the dynamic knee rig, a closed-loop identification method is needed where the input-output data is that of the system with the controller in a feedback loop (Ananth (1999)). Two methods for closed-loop identification can be found in literature: a direct method and an indirect method (Karimi (1998)). In this research, the indirect method is used as there is no access to the actual signal at the input of the process which is required for the direct method.

This paper presents the results of a closed-loop identification of a multivariable dynamic knee rig using the prediction error method ARMAX. A MIMO biomedical system, such as the dynamic knee rig, has multiple in- and outputs which can be interacting. However, as it is not known yet whether there are strong interactions between the variables, the system is treated as multiple single-input-single-output (SISO) systems for identification. Closed-loop identification is needed as the system is unstable in open-loop. An a priori known, simple controller is implemented to stabilize the system and to allow identification to be performed. From the obtained closed-loop identification, the system's model can be derived. Validation of the model is performed by comparing the simulated data with the measured data and is quantified by calculating the normalized root-mean-square error (NRMSE) value for each model, which is a measure for the goodness of fit.

This paper consists of the following sections. Section 2 introduces the multivariable dynamical knee rig. The closed-loop identification method is described in section 3. The results and validation of the identified models are presented in section 4. In section 5, a conclusion is drawn and future work is discussed.

2. DYNAMIC KNEE RIG

A dynamic knee rig is designed to impose a cyclic motion onto a post-mortem knee joint. This knee joint can be implanted with a knee prosthesis to evaluate its mechanical properties as a result of proper design, e.g. by measuring pressures under the prosthesis during cycling. This test is thereby used to gain an improved understanding of the impact of total knee arthroplasty. For instance is such insight gained through the comparison of the knee kinematics prior and following arthroplasty or from the evaluation of the intra-articular pressure distribution.

The system used in this study is shown in Figure 1. Three linear actuators are used in the system to impose movement to the knee joint. Biomedically speaking, the quadriceps force consists of 4 muscles from which 3 are responsible for extending the knee (i.e. vastus lateralis, vastus medialis and vastus intermedius) (Marieb (2011)). The resulting force of these 3 muscles is mimicked by linear actuator A. The position of the ankle joint in vertical direction (y-direction) is changed by actuator B, while the horizontal position (x-position) depends on actuator C. Characteristic for this system is that the kinematics (ankle position) and kinetics (applied quadriceps force) are fully disconnected.

In the dynamic knee rig, both post-mortem knee joints and mechanical knee joints can be placed. For this identification study a post-mortem knee joint was unnecessary, so a simplified mechanical knee joint D was placed in the system. The mechanical knee joint consists of a combination of three revolute joints (hip, ankle and knee) and two segments, i.e. the upper leg and the lower leg.

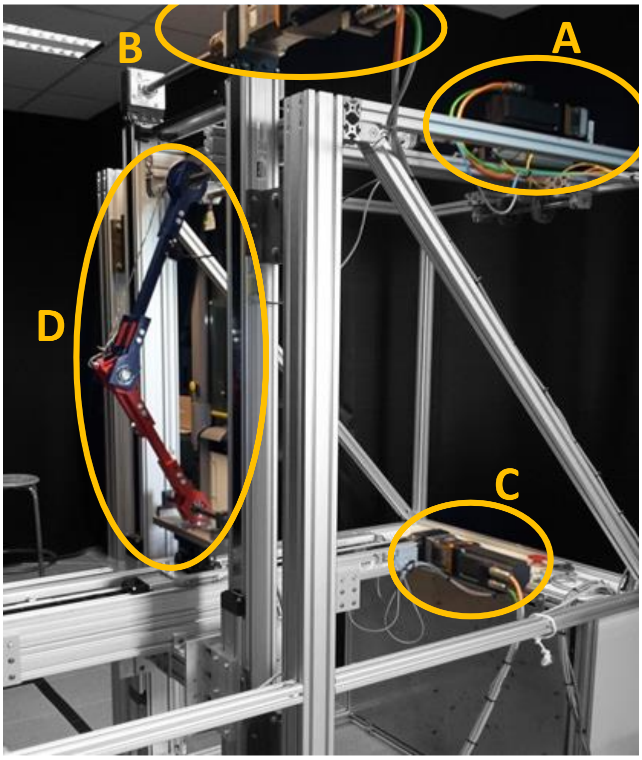

Fig. 1. Dynamic knee rig with 3 linear actuators (A,B, and C) and a mechanical knee (D).

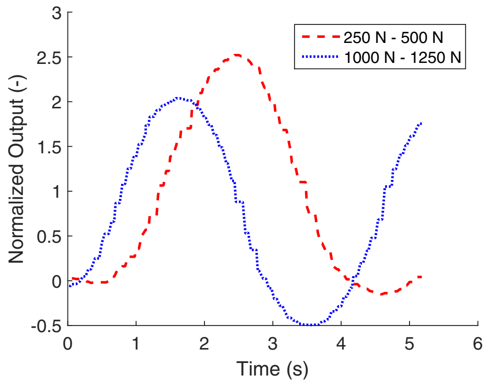

Fig. 2. Capturing the system's nonlinear dynamics.

The system has three inputs: the force reference to the internal control of linear actuator A which is set up as a force actuator and the position references to the internal control of linear actuators B and C which are set up as position actuators. The measured outputs of the system are the displacement of the ankle in the x- and y-direction and the force applied by linear actuator A, which is called the quadriceps force in the remainder of this paper. As these outputs are directly measured, they can be used in a feedback loop in order to control the system making this system a collocated MIMO system.

The system's dynamics for the quadriceps force are captured by a series of short step measurements in different ranges of the quadriceps force (Figure 2). Two steps of 250 N are given at opposite ends of the full force range as input and the normalized output is plotted. From these results, it is clear that the system is unstable resulting in highly oscillatory outputs. Figure 2 also shows the nonlinearity in the system as the output changes for each interval.

A MIMO dynamical system has interactions between input and output variables, i.e. giving an input signal to the x-variable can have an (unwanted) influence on the y-variable and visa versa. However, when these interactions are not strong, it is possible to neglect them and the MIMO system can be represented by a combination of multiple SISO systems. Taking this into account, the closed-loop dynamic knee rig can be represented by the block diagram shown in Figure 3 where C_i represents the implemented controller for the x- and y-direction and the quadriceps force Q and P_i represents the corresponding process model for the combination of the knee joint and the linear actuator. X^*, Y^* and Q^* are the reference values for the x- and y-position and the quadriceps force while X, Y and Q are the output values. It is assumed here that there is a negligible interaction between the quadriceps force and the x- and y-displacement in order to simplify the model.

3. CLOSED-LOOP IDENTIFICATION

To identify the closed-loop system the prediction error method (PEM) is used. From the identified closed-loop system, the process model will then be derived.

3.1 Prediction error method

PEM builds a mathematical model for dynamical systems based on input-output data, denoted respectively u and

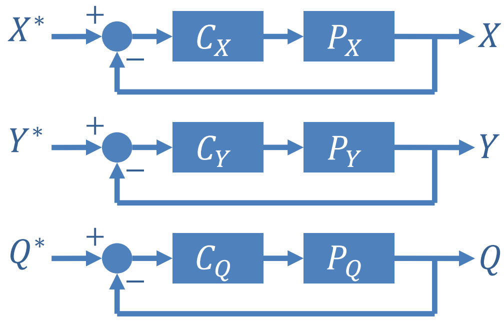

Fig. 3. Block diagram of the decentralized closed-loop control of the dynamic knee rig.

v. To explain the basic idea behind PEM let us denote $K^N = \{u(1), v(1), u(2), v(2), ..., u(N), v(N)\}$ all measured data up to time N. PEM can then be expressed in a few basic steps:

- The model of the system is described as a predictor for the output in the next time instance:
$$\hat{v}(t|t-1) = f(K^{t-1}) \tag{1}$$
where $\hat{v}(t|t-1)$ is a random 1-step-ahead predictor of $v(t)$ based on the data available at moment $t-1$ and f is an arbitrary function of observed data in the past.
- The predictor is parametrized in terms of a parameter vector θ which has a finite dimension:
$$\hat{v}(t|\theta) = f(K^{t-1}, \theta). \tag{2}$$
- An estimation of θ is determined based on the model parametrization and the data set K^N, so that the error between the estimation $\hat{v}(t|\theta)$ and the measured data $v(t)$ is minimal.

The general predictor model is given by (2). However, in this research a ARMAX model is applied which can be expressed by (3).

$$\begin{aligned} v(t) + a_1 v(t-1) + ... + a_{n_a} v(t-n_a) &= \\ b_1 u(t-n_k) + ... + b_{n_b} u(t-n_k-n_b+1) & \\ +c_1 e(t-1) + ... + c_{n_c}(t-n_c) + e(t) & \end{aligned} \tag{3}$$

with n_a the number of poles in the system, n_b the number of zeros plus 1, n_c the number of C coefficients and n_k the dead time in the system. $e(t)$ is a white-noise disturbance value. A more compact expression way of expressing equation (3) is

$$A(q)v(t) = B(q)u(t-n_k) + C(q)e(t) \tag{4}$$

with q the shift operator.

Using (3), the natural predictor becomes:

$$\begin{aligned} \hat{v}(t|\theta) = -a_1 v(t-1) - ... - a_{n_a} v(t-n_a) + & \\ b_1 u(t-n_k) + ... + b_{n_b} u(t-n_k-n_b+1) + & \\ c_1 e(t-1) + ... + c_{n_c} e(t-n_c) + e(t) & \end{aligned} \tag{5}$$

with

$$\theta = [a_1 \quad ... \quad a_n \quad b_1 \quad ... \quad b_{n_b} \quad c_1 \quad ... \quad c_{n_c} \quad 1]^T \tag{6}$$

which corresponds to

$$\hat{v}(t|\theta) = \theta^T \phi(t) \tag{7}$$

with

$$\begin{aligned}\phi(t) = [-v(t-1) \quad ... \quad -v(t-n_a) \quad u(t-n_k) \\ ... \quad u(t-n_k-n_b+1) \quad e(t-1) \quad ... \quad e(t-n_c) \quad e(t)].\end{aligned} \tag{8}$$

As the function of the dynamical system in this case is unknown, a rational transfer function will be used to identify the system, i.e. the system is treated as a linear black-box model. The unknown numerator and denominator polynomial coefficients will then be the parameters to find. The rational transfer function can be generally expressed in function of the shift operator (q^{-i}) with a delay of n_k samples as

$$\begin{aligned}G(q,\theta) = \frac{B(q)}{A(q)} = \\ \frac{b_1 q^{-n_k} + b_2 q^{-n_k-1} + ... + b_{n_b} q^{-n_k-n_b+1}}{1 + a_1 q^{-1} + ... + a_n q^{-n_a}}.\end{aligned} \tag{9}$$

3.2 Process model identification

In this research an indirect closed-loop identification method is used as the data at the input of the process is not available to measure. With the indirect closed-loop identification method, the closed-loop system for the quadriceps force is identified based on the reference input signals and the measured output. The closed-loop transfer function can be expressed as:

$$G_Q = \frac{Q^*}{Q} = \frac{C_Q P_Q}{1 + C_Q P_Q} \tag{10}$$

The open-loop transfer function can than be calculated as:

$$P_Q = \frac{G_Q}{C_Q(1 - G_Q)} \tag{11}$$

For the x- and y-displacement, similar reasoning can be used to obtain the open-loop transfer functions.

4. RESULTS AND VALIDATION

In this section the resulting transfer function models from the closed-loop identification are presented. Afterwards, the obtained models are validated by comparing measured data with simulation results.

4.1 Results

Based on insight into the system's instrumentation and dynamics, a simple proportional-derivative (PD) controller is implemented to stabilize the system with a K_p value of 1 and a T_d value of 0.05 for the x- and y- direction. For the quadriceps, the PD controller had a K_p value of 1 and a T_d value of 0.04.

For all three SISO loops closed-loop identification was performed using pseudo random binary signals (PRBS) as input signals. The input-output data is sampled with a sampling period of 0.06 s. The segment of the data used for identification is shown in Figure 4 for the x-direction. Similar input-output signals have been obtained for the quadriceps force and the y-direction.

The resulting identified closed-loop transfer functions from ARMAX are discrete time transfer functions. The resulting continuous time transfer functions for the x-direction, the y-direction and the quadriceps force are respectively given by (12), (13) and (14).

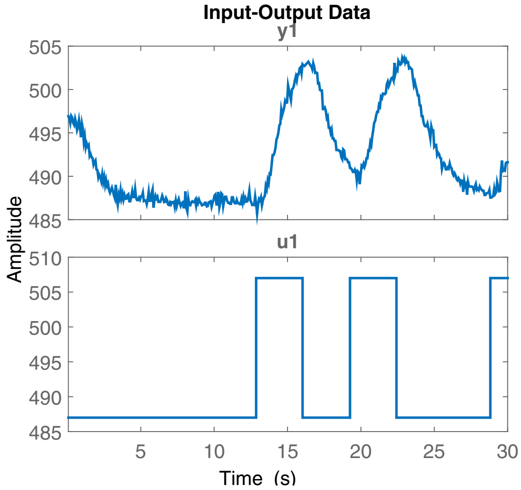

Fig. 4. Input-output data for the x direction.

$$G_X(s) = \frac{0.004s^3 - 3.29s^2 - 142.9s + 1189}{s^4 + 20.21s^3 + 1254s^2 + 2373s + 1189} \tag{12}$$

$$G_Y(s) = \frac{-0.05s^3 - 5.88s^2 - 194.2s + 1241}{s^4 + 24.23s^3 + 1414s^2 + 2056s + 1242} \tag{13}$$

$$\begin{aligned}G_Q(s) = \\ \frac{-0.67s^4 - 25.37s^3 - 194.6s^2 + 14720s + 117600}{s^5 + 65.06s^4 + 3704s^3 + 56710s^2 + 17680s + 104400}\end{aligned} \tag{14}$$

From the obtained transfer functions and the given PD controllers, the process model can be derived. The resulting open-loop transfer functions for the x-direction, the y-direction and the quadriceps force are respectively given by (15), (16) and (17).

$$P_X(s) = \frac{0.006(s - 20.99)(s - 7.41)}{(s + 2.052)(s - 0.00009)} \tag{15}$$

$$\begin{aligned}P_Y(s) = \\ \frac{0.0065(s - 24.85)(s - 5.533)(s - 0.000085)}{(s + 1.62)(s + 0.0005)(s - 0.000086)}\end{aligned} \tag{16}$$

$$\begin{aligned}P_Q(s) = \\ \frac{0.0022(s - 127.1)(s - 20.51)(s + 9.20)}{(s + 19.48)(s + 3.863)(s - 0.007901)}\end{aligned} \tag{17}$$

4.2 Validation

Validation of the models is obtained by comparing the output of the identified transfer function with the measured output signals. The measured output signals are the same data as for the identification. However, a different time segment is selected to obtain a qualitative validation, i.e. between 30 and 40 seconds. In order to obtain the output of the identified models, a simulation in the MATLAB/SIMULINK environment is performed using the designed PRBS input signal and selecting the corresponding time segment. The resulting simulated signal is plotted together with the measured output in Figure 5 for the x-direction. Similar signals have been obtained for the y-direction and the quadriceps force.

To compare quantitatively the fit of the model to the real measurements the normalized root-mean-square error

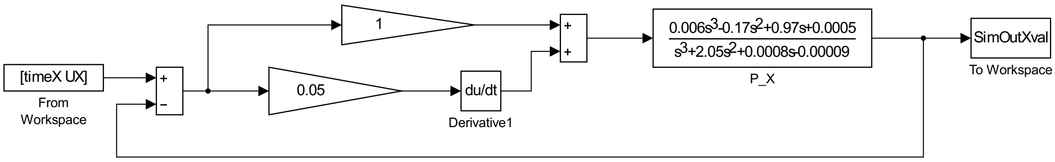

Fig. 6. Simulink scheme for the x-direction. Similar schemes are made for the y-direction and the quadriceps force.

(NRMSE) value is calculated for each model. The NRMSE value for each model is obtained using:

$$NRMSE = 100\left(1 - \frac{||m(t) - n(t)||}{||m(t) - \text{mean}(m(t))||}\right) \qquad (18)$$

with $m(t)$ the measured output, $n(t)$ the simulated output and 'mean(.)' the mean value of a data vector.

The resulting NRMSE values for each identified model are presented in Table 1.

Table 1. NRMSE for all identified models.

	NRMSE value (%)
x-direction	87.33
y-direction	78.40
Quadriceps force	86.66

Note that the y-direction has a decreased fit which can be explained by the noise on the measurement equipment. There is more noise in the y-direction as the knee rig has a bigger range of movement in the y-direction compared to the x-direction while it is using the same measurement equipment. This will result in an increased noise in the y-direction compared to the x-direction.

4.3 Experimental results

The identified models are used in an experiment with a step input to compare the output of the measured signals with that of the identified models. For each of the three models a step is given as reference signal to the closed-loop system and the resulting output is measured.

The output signals of the identified models are obtained using the MATLAB/SIMULINK environment where the same input as that of the measurements is applied to the closed-loop system. Figure 6 shows the block scheme for the x-direction process. Similar block schemes are used to perform the simulations for the y-direction and the quadriceps force.

Figures 7, 8 and 9 show the simulated output, the measured output and the given input signal for respectively the x-direction, the y-direction and the quadriceps output.

The respective NRMSE values for these experiments are also calculated to give a quantitative measure of the goodness of fit of the identified models in comparison to the measured experimental outputs. The resulting values for the NRMSE of the measurement of the x-direction, the y-direction and the quadriceps force are respectively 49.54 %, 71.13 % and 86.68 %.

4.4 Discussion

The validation results shown in Figure 5, indicate a correspondence between the identified model and the real-life system. The goodness of fit between the simulated outputs and the measured outputs has been expressed in NRMSE values presented in Table 1. The resulting NRMSE values suggest that the identified models for the x-direction, the y-direction and the quadriceps force are adequate. However, improvement of the model is still possible when interaction between the variables is taken into account. Also disturbances and noise can be an explanation for deviation in the results as they are not taken into account in the current identification.

Even though the obtained models differ from the real system, they can be used to design a suitable control

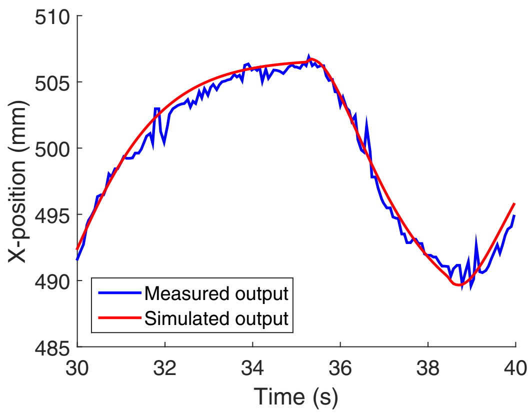

Fig. 5. Simulated output vs measured output.

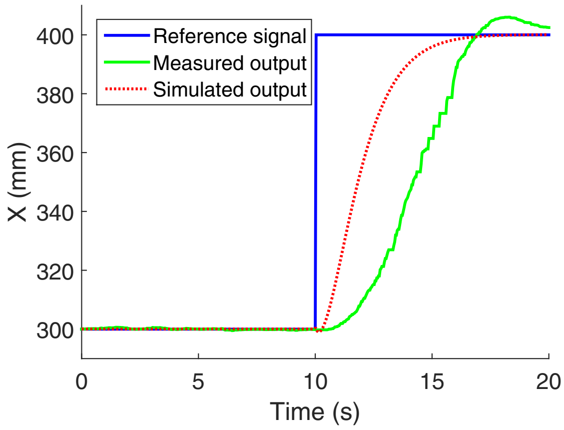

Fig. 7. Validation of the model for the x-direction.

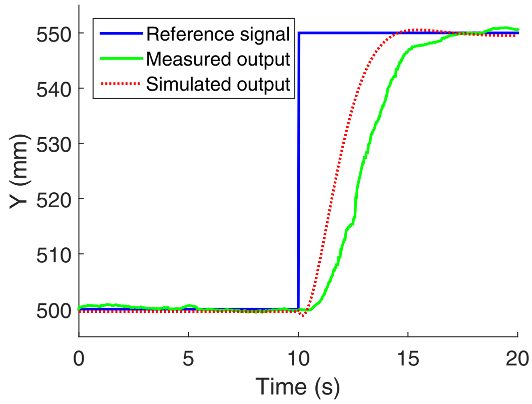

Fig. 8. Validation of the model for the y-direction.

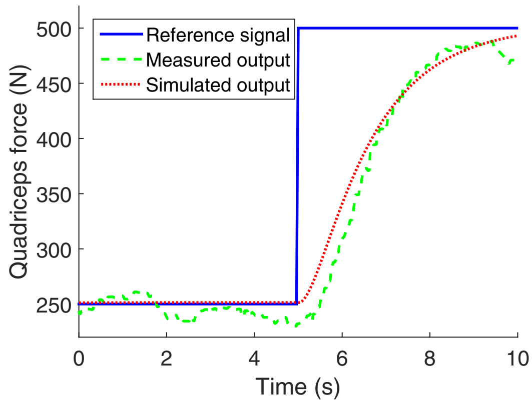

Fig. 9. Validation of the model for the quadriceps force.

strategy. Robustness requirements have to be taken into account during the design process of the controllers in order to deal with the model uncertainties (clearly visible in Figures 7 and 8). The identified models will be used in a model-based control design technique in order to enhance the current control performance of the system. For each of the three variables, a PID controller will be designed based on the identified model in order to control the system's dynamics and apply natural movements on the knee joint under observation. In this way, insight into the dynamics of diseased knee joints or design improvements for knee prosthetics will be obtained.

5. CONCLUSION

This paper presents closed-loop identification of a multiple-input/multiple-output (MIMO) dynamical knee rig. It is assumed that there is no interaction between the different system outputs resulting in the possibility of treating the MIMO system as a combination of several single-input/single-output (SISO) systems. A prediction error method ARMAX is used to obtain the closed-loop transfer function from which the process model is derived based on the feedback law. The results are validated by comparing simulated output with measured responses of the dynamic knee rig. To quantify the validation of the models, a NRMSE value is calculated for each subsystem. The identified models are then used to compare measured outputs with experimental outputs for a step input. The results show that for all three subsystems, the NRMSE values are sufficiently high to have a validation of the closed-loop identified models. However, improvement in the models is possible when taking into account interaction between the variables of the MIMO system when deriving the open-loop models. Future work will focus on defining the interaction between these variables and using the obtained models to design a suitable control strategy for the system.

REFERENCES

Ananth, I. and Chidambaram, M. (1999). Closed-loop identification of transfer function model for unstable systems. *Journal of the Franklin Institute*, 336(7), 1055-1061.

Bourne, R., Goodfellow, J.W. and O'Connor, J.J. (1978). A functional analysis of various knee arthroplasties. *Trans. Orthopaedic Research Society*, 24, 156–160.

Box, G.E.P. and Jenkins, D.R. (1970). *Time series analysis, forcasting and control.* Holden-Day, San Francisco.

Chevalier, A., Ionescu, C.M. and De Keyser, R. (2014). Nonlinear Predictive Control of Shank Movement Around the Knee Joint. *19th World Congress of The International Federation of Automatic Control*, Cape Town, South Africa August 24-29, 9265–9270.

De Loës, M., Dahlstedt, L.J. and Thomée, R. (2000). A 7-year study on risks and costs of knee injuries in male and female youth participants in 12 sports. *Scandinavian Journal of Medicine & Science in Sports*, 10(2), 90–97.

Fisher, R.A. (1912) On an absolute criterion for fitting frequency curves. *Messenger of Mathematics*, 41, 155.

Karimi, A. and Landau I.D. (1998) Comparison of the closed-loop identification methods in terms of the bias distribution. *Systems & Control Letters*, 34, 159-167.

Majewski, M. (2006). Epidemiology of athletic knee injuries: A 10-year study. *The Knee*, 13(3), 184–188.

Maquet, P.G.J. (1984). *Biomechanics of the knee with application to the pathogenesis and the surgical treatment of osteoarthritis.* Springer-Verlag, Berlin Heidelberg, 2nd edition.

Marieb, E.N. and Hoehn, K. (2011). *Anatomy & Physiology.* Pearson Education, San Francisco, 4th edition.

OECD (The organisation for economic co-operation and development). Health at a Glance: Europe 2012. *http://www.oecd-ilibrary.org.* Accessed October 2013.

OECD (2014). Organisation for Economic Co-operation and Development (OECD), "Ageing Populations : High Time for Action", http://www.oecd.org/employment/emp/34600619.pdf, accessed April 2014.

Pai, P.F. (2013). Time-frequency analysis for parametric and non-parametric identification of nonlinear dynamical systems. *Mechanical Systems and Signal Processing*, 36(2), 332-353.

Pitkin, M.R. (2010). *Biomechanics of lower limb prosthetics.* Springer-Verlag, Berlin Heidelberg.

Shaw, J.A. and Murray, D.G. (1973). Knee joint simulator, *Clinical Orthopaedics and Related Research*, 94, 15–23.

Surgery Price. Knee Replacement Price. *http://www.surgeryprice.co.uk.* Accessed October 2013.

Zavatsky, A.B. (1997). A Kinematic-freedom analysis of a flexed-knee-stance testing rig. *Journal of Biomechanics*, 30(3), 277-280.